



### About the Lesson

In this activity, students will use a CBR 2 to make a height-time plot of a bouncing ball. They will derive two forms of a quadratic equation to match the height-time graph of one of the bounces. As a class, students will explore the reasons for the similarities of their equations.

### Vocabulary

- vertex
- zeroes of a graph
- x-intercept
- coefficient
- absolute value
- quadratic equation

### Teacher Preparation and Notes

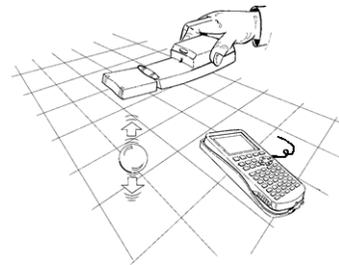
- Avoid using tennis balls. The furry exterior interferes with the signal that the CBR 2 uses. Paddle balls, basketballs, or playground balls work well.
- Decide beforehand if each student is to obtain a graph, or if students are to work in groups. When one calculator has the data, it can be sent calculator to calculator using the mini-USB link cables until each one in the group has the data.

### Activity Materials

- Compatible TI Technologies:
  - TI-84 Plus\*
  - TI-84 Plus Silver Edition\*
  -  TI-84 Plus C Silver Edition
  -  TI-84 Plus CE

\* with the latest operating system (2.55MP) featuring MathPrint™ functionality.

- CBR 2™ motion sensor unit with mini-USB connecting cable
- Vernier EasyData® App
- Paddleballs or basketballs
- A level non-carpeted floor



### Tech Tips:

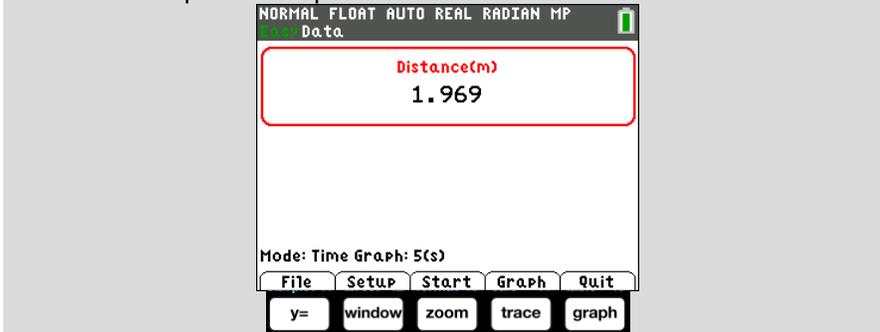
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

### Lesson Files:

- Curve\_Ball\_Student.pdf
- Curve\_Ball\_Student.doc



**Tech Tip:** While using the EasyData app, the tabs at the bottom of the screen indicate menus that are accessed by pressing the key directly below it. A frequent example is shown:

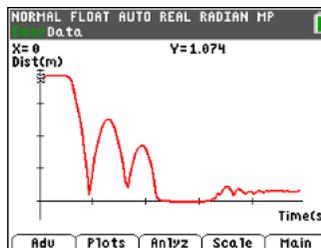
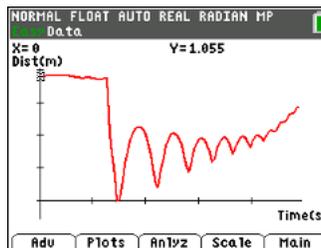


### Introduction

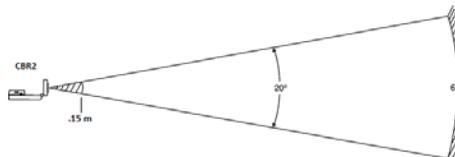
You use the formula  $A = x^2$  to find the area of a square whose side is  $x$ . This is a quadratic relation between the side length and area of the square since the variable  $x$  is raised to the second power. There are many examples of quadratic relations in the real world, including height and time variations for a bouncing ball.

### Teaching Notes:

- The warning given to students about holding the CBR 2 at a constant height above the floor is important. The graph at the right (top) is similar to one made by a student who lowered the CBR 2 with the ball. The ball in the graph at the right (bottom) travelled in and out of the “beam” of the CBR 2.



- The path of the CBR 2 beam is not a narrow, pencil-like beam, but fans out in all directions up to  $10^\circ$  in a cone-shaped beam. The CBR 2 can handle distances from 0.15 meters to about 6.0 meters.



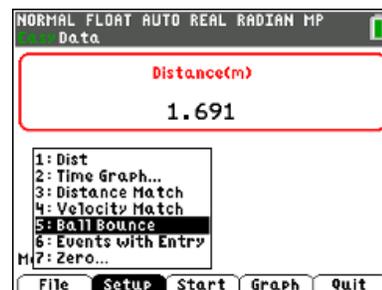


- If students have trouble obtaining good results, try using a larger ball.
- To expedite data collection, use three students: one to hold the CBR 2, one to release the ball, and one to run the calculator. Depending on your students, the activity can be done in pairs.
- For best results, hold the sides of the ball, and then quickly move your hands outward to release the ball.
- The value of A found in the activity is a constant since it is approximately one-half of acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$ .

### Collecting the Data

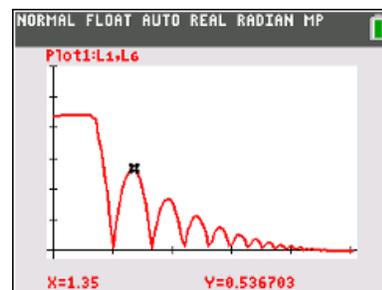
Students will Change the **Setup** and select Ball Bounce.

The mode is Ball Bounce. Students will hold the CBR 2 at about one meter above the floor and the ball no closer than 0.15 meters (6 inches) below the CBR 2. Then they may disconnect the CBR 2 to collect the data. When you reconnect to the calculator, the CBR 2 transfers the data to the calculator.



1-2. Sample data for bounce one is shown to the right.

- Use the arrow keys to move through your height-time plot. Choose **one** of your bounces, and find the coordinates of the vertex and the x-intercepts. Record them below. Make a sketch of your graph to the right.



**Student answers will vary.**

Sample response:

Vertex (h, k)	x-intercepts (zeroes)
h = 1.35    k = 0.537	1.0    and    1.65



**Tech Tip:** After students record the vertex and x-intercepts, they will need to exit the EasyData app. Make a note of the message that appears. The data in L6 is the height, L7 is the velocity, and L8 is the acceleration. Also the graph of L1 vs L6 is defined in Plot1.

It is also important to know which lists contain data if your students are doing the Going Further questions. The last one uses the QuickFit&Plot-EQ feature that will clear out lists and changes the Plot. See the response for question 3 in Going Further.



### Looking at the Results

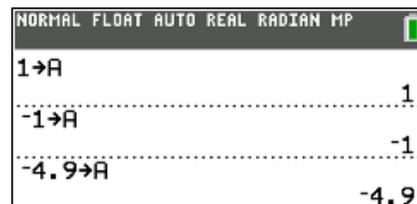
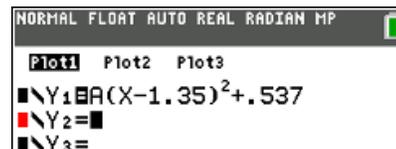
- To find the value for A, start the *Guess-and-Check* method by storing 1 into the variable A on the home screen (1[sto→][alpha]A). Check your guess by pressing [graph]. Store a new value to A and check your guess. Keep experimenting until you get a good fit. Record your equation here.

**Student answers will vary.**

Sample response:  $y = -4.9(x-1.35)^2 + 0.537$

*Teaching Note: The guess-and-check method is an effective way for students to learn the relationship between the leading coefficient of a quadratic and the shape of the graph of the parabola. Students should arrive at approximately -4.9 for the value of A. The equation for the sample data that matches the first bounce:*

$$y = -4.9(x-1.35)^2 + 0.537$$



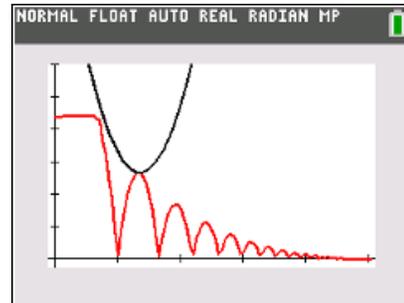


2. Describe the relationship of the value of A with the shape of the parabola. (What happens when A changes from negative to positive? What happens if you increase the absolute value of A?)

**Student answers will vary.**

Sample response: When A is positive, the parabola is curved upward with the vertex as a minimum. When A is negative, the parabola is curved downward with the vertex as a maximum. When the absolute value of A increases, the width of the parabola decreases. When the absolute value of A decreases, the width of the parabola increases.

A=1 in the graph to the right. So the student needs to make A negative and increase the absolute value of A to make the parabola more narrow.



3. Another form of quadratic equation uses the x-intercepts or zeroes of the equation. Using the x-intercepts that you recorded, put the equation into Y2:

$$y = A(x - \quad)(x - \quad)$$

What do you notice about the resulting graph?

**Student answers will vary.**

Sample response: The resulting graph of the factored form of the parabola  $[y=-4.9(x-1)(x-1.65)]$  is the same as the vertex form.



**Tech Tip:** To help students more easily see Y2 being graphed, arrow to the leftmost icon and select the settings shown. The icon that looks like a key will show the path of the function and will allow students to see that Y1 and Y2 are equivalent.



4. What value for A did your classmates use? Explain what you discovered.

**Student answers will vary.**

Sample response: The other student groups should have arrived at  $-4.9 = A$ . This is  $\frac{1}{2}(g)$  where  $g$  is the acceleration due to gravity. Gravity accelerated the balls that were dropped by each group.

5. Find an equation in either form for the next bounce to the right of the bounce you used previously. Record here.

**Student answers will vary.**

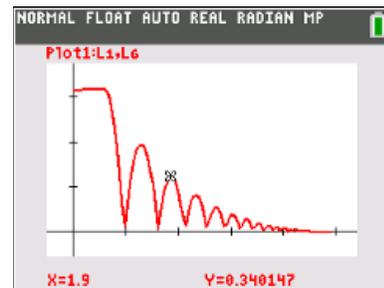
Sample response:

$$y = -4.9(x - 1.9)^2 + 0.3402 \quad \text{Vertex form}$$

$$y = -4.9(x - 1.65)(x - 2.2) \quad \text{Factored form}$$

How does this equation compare with the equation for the previous bounce in question 1 or 3?

Sample response: For the bounce to the right, the only difference in the equation will be the vertex or the x-intercepts. For the sample data:



### Going Further

1. If a ball that was more or less bouncy was used instead, how would it affect the value of A in the equation?

Sample response: The value of A will still be  $\frac{1}{2}(g) = -4.9 \text{ m/s}^2$ .



2. The graph of the vertex form of the parabola matched the graph of the equation using the x-intercepts. Give an algebraic reason why these two forms of a parabola are equivalent equations.

Sample response:

$$-4.9(x-1.35)^2+0.537=-4.9(x-1)(x-1.65)$$

$$-4.9(x^2-2.7x+1.8225)+0.537=-4.9(x^2-2.65x+1.65)$$

$$-4.9x^2+13.23x-8.39325=-4.9x^2+12.985x-8.085$$

Allowing for experimental variation, these two quadratic expressions are essentially the same after expanding each one to standard form.

3. What quadratic equation does the calculator give to match your first bounce?

**Student answers will vary.**

Sample response: To do the quadratic regression on the first bounce would require that the data outside the first bounce be deleted. It is less complicated to do a Quick Plot & Fit-EQ that requires a minimum of three points to calculate a quadratic regression that is given in standard form. **The directions are in the student activity.** Pay close attention to the settings; you want to **avoid** putting the QuickPlot&Fit-EQ data in L1, L6, Y1, Y2, and Plot1. Then the graph of the original ball bounce data does not disappear from the graph when the QuickPlot&Fit-EQ is stored.

